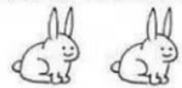
Nov. 2^{nd}, 2021

Quiz & Optimization Problems

Suppose you have one rabbit.

Now suppose someone gives you one more rabbit.



Now, if you count your rabbits, you have two rabbits. So one rabbit plus one rabbit equals two rabbits. So one plus one equals two.

$$1 + 1 = 2$$

And that is how arithmetic is done.

Now that you understand the basic idea behind arithmetic, let's take a look at a simple easy-to-understand example that puts into practice what we just learned.

Try It Out
Example 1.7

$$\log \Pi(N) = \left(N + \frac{1}{2}\right) \log N - N + A - \int_N^\infty \frac{\overline{B}_1(x)\mathrm{d}x}{x}, \quad A = 1 + \int_1^\infty \frac{\overline{B}_1(x)\mathrm{d}x}{x}$$

$$\log \Pi(s) = \left(s + \frac{1}{2}\right) \log s - s + A - \int_0^\infty \frac{\overline{B}_1(t)\mathrm{d}t}{t + s}$$

$$\begin{aligned}
\log \Pi(s) &= \lim_{n\to\infty} \left[s \log(N+1) + \sum_{n=1}^{N} \log n - \sum_{n=1}^{N} \log(s+n) \right] \\
&= \lim_{n\to\infty} \left[s \log(N+1) + \int_1^N \log x \,\mathrm{d}x - \frac{1}{2} \log N + \int_1^N \frac{\overline{B}_1(x)\mathrm{d}x}{x} \right. \\
&\quad - \int_1^N \log(s+x)\,\mathrm{d}x - \frac{1}{2}[\log(s+1) + \log(s+N)] \\
&\quad \left. - \int_1^N \frac{\overline{B}_1(x)\mathrm{d}x}{s+x} \right] \\
&= \lim_{n\to\infty} \left[s \log(N+1) + N \log N - N + 1 + \frac{1}{2} \log N + \int_1^N \frac{\overline{B}_1(x)\mathrm{d}x}{x} \right. \\
&\quad - (s+N)\log(s+N) + (s+N) + (s+1)\log(s+1) \\
&\quad \left. - (s+1) - \frac{1}{2}\log(s+1) - \frac{1}{2}\log(s+N) - \int_1^N \frac{\overline{B}_1(x)\mathrm{d}x}{s+x} \right] \\
&= \left(s + \frac{1}{2}\right) \log(s+1) + \int_1^\infty \frac{\overline{B}_1(x)\mathrm{d}x}{x} - \int_1^\infty \frac{\overline{B}_1(x)\mathrm{d}x}{s+x} \\
&\quad + \lim_{n\to\infty} \left[s \log(N+1) + \left(N + \frac{1}{2}\right) \log N \right.
\end{aligned}$$

Every STEM class...

Quiz 5.

problem 1. Sketch the graph of the function $f(x) = x^2 + \frac{1}{x} - 1$.

domain: $(-\infty, 0) \cup (0, +\infty)$.

$f'(x) = 2x - \frac{1}{x^2} = 0$ $\quad 2x^3 = 1 \rightsquigarrow x^3 = \frac{1}{2}$, $\quad x = \sqrt[3]{\frac{1}{2}}$. critical pt.

$f(x) = \sqrt[3]{\frac{1}{4}} + \frac{1}{\sqrt[3]{\frac{1}{2}}} - 1 = \frac{1}{\sqrt[3]{4}} + \sqrt[3]{2} - 1 > 0$

$x < 0$: $f'(x) < 0$, f decreasing in $(-\infty, 0)$.

$\sqrt[3]{\frac{1}{2}} > x > 0$: $f'(x) < 0$, f decreasing in $(0, \sqrt[3]{\frac{1}{2}})$.

$x > \sqrt[3]{\frac{1}{2}}$: $f'(x) > 0$, f increasing in $(\sqrt[3]{\frac{1}{2}}, +\infty)$.

$\rightsquigarrow (\sqrt[3]{\frac{1}{2}}, \frac{1}{\sqrt[3]{4}} + \sqrt[3]{2} - 1)$ is the local minimum

$f''(x) = 2 + \frac{2}{x^3} = 0$, $\quad x = -1$.

$x > 0$: $f''(x) > 0$ $\quad$ up $\quad (0, +\infty)$.

$x < -1$: $f''(x) > 0$ $\quad$ cocave up. in $(-\infty, -1)$

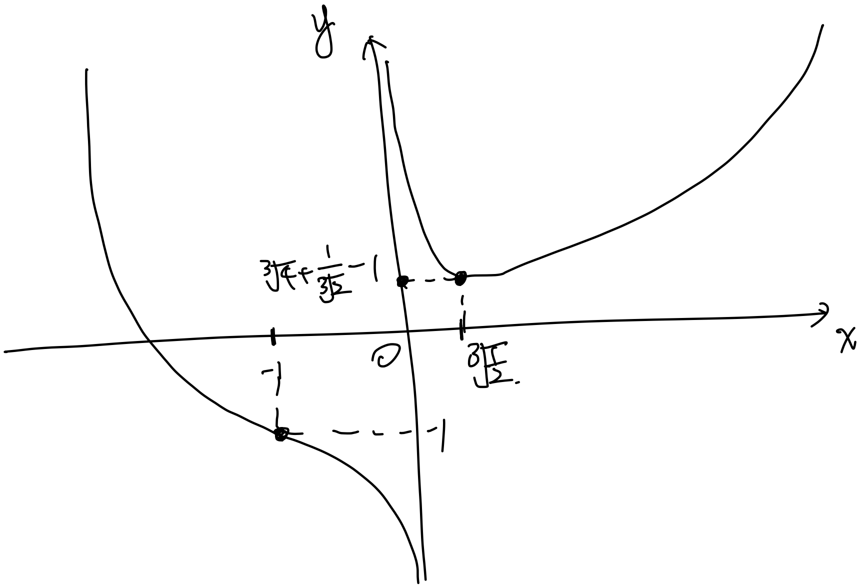
$x > -1$: $f''(x) < 0$ $\quad$ down in $(-1, 0)$

Quiz 5.

problem 1. Sketch the graph of the function $f(x) = x^2 + \frac{1}{x} - 1$.

horizontal asymptote: NO, $\lim_{x \to \pm\infty} \left(x^2 + \frac{1}{x} - 1 \right) = +\infty$.

vertical: $\lim_{x \to 0^-} f(x) = -\infty$, & $\lim_{x \to 0^+} f(x) = +\infty$.



Warning: Inflection pt should first lie in the domain of f.

Problem 2. (8 Points) Let R be a rectangle with length a and side b, so that $\frac{a}{b} = \frac{4}{3}$. Let d be the length of the diagonal. If d increases at the rate of 2cm/s, what's the rate that the area A of R grows when $d = 5$cm?

$D^2 = a^2 + b^2$, so $D = \sqrt{a^2+b^2}$. $\frac{dD}{dt} = 2$ cm/s. WANT $A'(t)$. when $D = 5$cm.

$A(t) = a(t) \cdot b(t)$; $A'(t) = a'(t)\, b(t) + a(t) \cdot b'(t)$.

$\frac{a}{b} = \frac{4}{3}$. $\rightsquigarrow$ $\boxed{a(t) = \frac{4}{3} b(t)}$ then $D(t) = \sqrt{(a(t))^2 + (b(t))^2} = \sqrt{\frac{16}{9}(b(t))^2 + (b(t))^2} = \sqrt{\frac{25}{9}(b(t))^2}$

$= \frac{5}{3} b(t)$. $\rightsquigarrow$ $D'(t) = \frac{5}{3} b'(t)$ $\rightsquigarrow$ $b'(t) = 1.2$ cm/s. $a'(t) = \frac{4}{3} b'(t) = 1.6$ cm/s.

$D'(t) = 2$ cm/s

$D(t) = 5$ cm $\rightsquigarrow$ $b(t) = \frac{3}{5} D(t) = 3$ cm, $a(t) = \frac{4}{3} b(t) = 4$ cm.

Finally. $A'(t) = a'(t) \cdot b(t) + a(t) \cdot b'(t) = 1.6 \times 3 + 4 \times 1.2 = 0.96$ cm^2/s.

Problem 2. (8 Points) Let R be a rectangle with length a and side b, so that $\frac{a}{b} = \frac{4}{3}$. Let d be the length of the diagonal. If d increases at the rate of 2cm/s, what's the rate that the area A of R grows when $d = 5$cm?

$\boxed{D(t) = \frac{5}{3} b(t)}$ take derivative. w.r.t. t.

$D'(t) = \frac{5}{3} b'(t).$ $D'(t) = 2$ cm/s.

$\frac{5}{3} \underline{b'(t)} = 2$ $b'(t) = 2 \times \frac{3}{5} = \frac{6}{5} = 1.2$ cm/s

$a(t) = \frac{4}{3} b(t).$ $a'(t) = \frac{4}{3} b'(t) = \frac{4}{3} \times 1.2 = 4 \times 0.4 = 1.6$ cm/s.

$D(t) = 5$ cm. $b(t) = \frac{3}{5} D(t) = \frac{3}{5} \times 5$ cm $= 3$ cm.

$a(t) = \frac{4}{3} b(t) = \frac{4}{3} \times 3$ cm $= 4$ cm.

Optimization Problem.

43. (a) If $C(x)$ is the cost of producing x units of a commodity, then the **average cost** per unit is $c(x) = C(x)/x$. Show that if the average cost is a minimum, then the marginal cost equals the average cost.

(b) If $C(x) = 16{,}000 + 200x + 4x^{3/2}$, in dollars, find (i) the cost, average cost, and marginal cost at a production level of 1000 units; (ii) the production level that will minimize the average cost; and (iii) the minimum average cost.

marginal cost $\to C'(x)$.

average cost is a minimum $\to c'(x) = 0$

$$\left(\frac{C(x)}{x}\right)' = 0$$

$$\frac{xC'(x) - C(x)}{x^2} = 0$$

$$xC'(x) - C(x) = 0$$

$$\underbrace{C'(x) = \frac{C(x)}{x} = c(x).}$$

(i) $C(1000) = 200 + 6\sqrt{1000}^3 = 200 + 60000\sqrt{10}$

$c(1000) = 16 + 200 + 40\sqrt{10} = 216 + 40\sqrt{10}$

$C'(1000) = 200 + 60\sqrt{10}$.

$\underbrace{C'(x) = 200 + 6x^{\frac{1}{2}}}$.

$\underbrace{c(x) = \frac{C(x)}{x} = \frac{16000}{x} + 200 + 4\sqrt{x}}$.

$c'(x) = -\frac{16000}{x^2} + \frac{2}{\sqrt{x}}$.

(ii) $x_0 =$

$c'(x) = 0 \rightsquigarrow \frac{2x^{\frac{3}{2}} - 16000}{x^2} = 0 \rightsquigarrow x_0^{\frac{3}{2}} = 8000, \quad x_0 = \sqrt[3]{(8000)^2} = (20)^2 = 400$

$x^{\frac{3}{2}} \to$ increasing.

x_0 is a minimum: $x < 400$: $c'(x) < 0$, $x > 400$, $c'(x) > 0$, so 400 is a (local) minimum of c.

(iii) $c(x_0) = 40 + 200 + 4 \cdot 20 = 320$

Anti-derivative.

$F'(x) = f(x)$, then F is the **anti-derivative**.

Find f: 1) $f''(\theta) = \sin\theta + \cos\theta$, $f(0)=3$, $f'(0)=4$; 2) $f'(t) = 1.5\sqrt{t}$, $f(4)=10$.

$(\cos\theta)' = -\sin\theta$

$(-\cos\theta)' = \sin\theta$.

$(\sin\theta)' = \cos\theta$.

$f'(\theta) = -\cos\theta + \sin\theta + C$, $f'(0)=4$.

$f'(0) = -1 + 0 + C = 4$, $C = 5$

$f'(\theta) = -\cos\theta + \sin\theta + 5$.

$f(\theta) = -\sin\theta - \cos\theta + 5\theta + C$.

$f(0) = -1 + C = 3$, $C = 4$.

$f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$.

$(x^a)' = a x^{a-1}$.

antiderivative of x^{a-1}

is $\frac{1}{a} x^a$.

Reminder:

- Quiz 6: Optimization (16 pts) + Antiderivative (8 pts). ← 2 prbs?

  practice problems: problem 27, 31, 41 and 42 in Chpt 3 Review

- OH today 1-2pm, appointments accepted.

See U On Thursday!